Mitigating Short-Delay Multipath: a Promising New Technique

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BIOGRAPHY

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ABSTRACT

In the recent years, several advanced signal processing techniques have been devised to mitigate errors induced by multipath signals. They have proved very efficient against multipath having a medium or large delay with respect to the direct signal. Typically, the errors are largely removed for multipath delays higher than around 20 m. From a theoretical point of view, this constitutes a dramatic improvement with respect to simpler techniques such as the narrow correlator.

However, when analysing field data, the resulting performance is often disappointing: the improvement over the narrow-correlator is marginal. The reason is that most of the multipath signals enter the receiver with a short delay with respect to the direct signal, rendering the mitigation scheme ineffective.

This paper presents a new signal processing algorithm for multipath mitigation, based on the a-posteriori estimation of the tracking errors. This method is shown to achieve up to 50% better immunity to short-delay multipath compared to other state-of-the-art techniques. The performance is evaluated both from a theoretical analysis and from real data. A comparison with state-of-the-art technologies demonstrates an improved mitigation capability in most practical situations.

INTRODUCTION

As the dominant error source in high-accuracy GPS applications, multipath has received considerable interest for many years. A number of signal processing techniques have been devised for mitigating the multipath errors on the satellite range measurements. Many of those rely on modifying the tracking loop discriminator so as to make it resistant to multipath signals. One of the best-known techniques is the narrow correlator that has been introduced in the early nineties [6] and led to a significant reduction of the peak multipath error on the code-phase measurement. State-of-the-art techniques include the Strobe and Edge Correlators [9], the High Resolution Correlator (HRC) [4] or the Gated Correlator [3]. They all achieve similar performance figures, though by different means. Another category of techniques relies on an estimation of the parameters (amplitude, delay and phase) of the line-of-sight (LOS) signal along with those of all the multipath components. It includes the Multipath Estimating Delay Lock Loop (MEDLL) [8], or the Modified Rake DLL (MRDLL) [2]. These latter techniques achieve comparable level of performance as the previous ones at the expense of a higher computational burden.

All those techniques suffer from the common drawback that they are ineffective against multipath signals arriving with a short delay (less than about 20 m) with respect to the LOS signal. This is a strong limitation since real-life multipath tends to be of close-in, short-delay type. For example, it was reported in [1] that multipath analysis at the WAAS reference stations showed no better results for a MEDLL processing than for the older and simpler narrow correlator, due to the close-in nature of the multipath at these stations.

The motivation for the work that led to this paper has been the above shortcomings of the current state-of-the-art. The starting point has been the well-known property that the signal amplitude measurement (reported as the C/N0 by GPS receivers) is highly correlated with the multipath error in the code-phase measurement. This property is attractive for short-delay multipath, as the sensitivity of the signal amplitude to multipath is maximized for the short-delays.

This paper describes the patent-pending A-Posteriori Multipath Estimation (APME) technique, which relies on an a-posteriori estimation of the multipath error affecting the code tracking. Specifically, the tracking is done in a conventional narrow correlator DLL, offering a low tracking noise. The multipath error affecting the narrow correlator tracking is estimated in an independent module on the basis of different signal amplitude measurements. Subtracting this estimate from the code-phase measurement yields a substantial reduction of the error, especially for short-delay multipath.

As an introduction, this paper first describes the behavior of a traditional narrow correlator DLL in presence of multipath. The corresponding multipath error in the code-phase measurement will be derived. Later, the rationale and principle of operation of the APME technique for estimating this error will be discussed. Performance figures will be derived both from simulations and from field results.

MULTIPATH ERROR IN NARROW DLL

For the reader not familiar with the subject, this section provides a description of the traditional code tracking process, and its response to multipath perturbations. Fig. 1 illustrates the different building blocks that are part of a conventional coherent Delay Lock Loop (DLL).

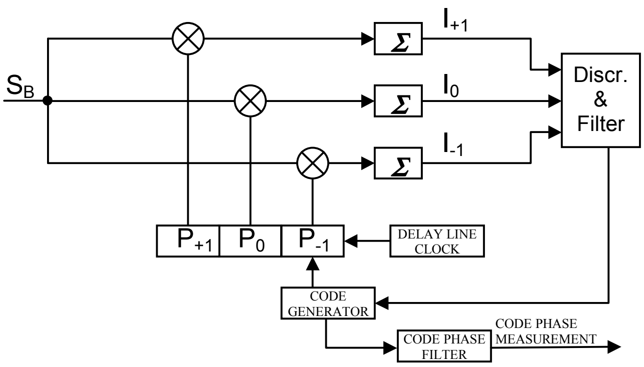

Fig. 1 Conventional coherent DLL implementation.

A local code replica is generated in the code generator at a rate that is continuously controlled by the loop discriminator and filter. The code enters a delay line where three different replicas are generated: P_0 is the punctual code replica, which has to be kept aligned with the incoming code. P_{-1} is the early replica, which is advanced by a fraction of a chip with respect to P_0, and P_{+1} is the late replica, delayed by a fraction of a chip with respect to P_0. The delay between the taps in the delay line is the inverse of the delay line clock frequency, and is traditionally referred to as "*d*/2" in units of code chips. For the GPS C/A-code, the code chip is close to 1 µs, and the chip length close to 293 m. The delay between the early and late taps is an important design parameter, which is referred to as the early-late spacing, and noted *d*. Many receivers use a so-called "Wide spacing" of *d* = 1 code chip. The spacing is said to be narrow if *d* is lower than 1 chip. It has been demonstrated that using a narrow spacing is beneficial both for multipath mitigation and tracking noise reduction. Namely, the maximum multipath error and the noise variance are proportional to *d* [6].

The multipliers followed by the accumulators yield the correlation between the baseband signal S_B and each of the local code versions. The resulting three correlations I_0, I_{-1} and I_{+1} are represented in Fig. 2 as a function of the delay misalignment $\Delta\tau$ between the incoming code and the local punctual code (P_0). When $\Delta\tau$ is positive, the local punctual code is late with respect to the incoming code, and the early correlation (I_{-1}) is higher than the late correlation (I_{+1}). The opposite occurs when $\Delta\tau$ is negative. As can be expected, the correlation between the punctual and the incoming code (I_0) reaches its peak when they are aligned, i.e. when $\Delta\tau = 0$.

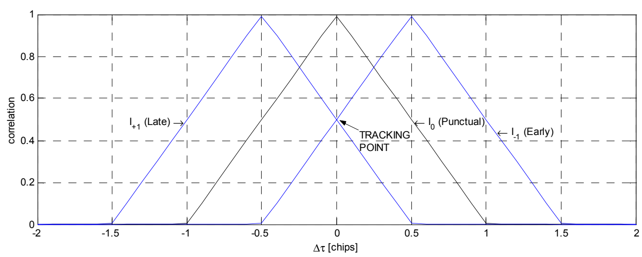

Fig. 2 Normalized early, punctual and late correlation values.

The role of the DLL is to keep the punctual code P_0 aligned with the incoming code. It can be seen in the figure that it is achieved if the two side correlations I_{-1} and I_{+1} are equal. Therefore, the DLL is made such that it adapts the frequency of the local code such that the equality $I_{-1}=I_{+1}$ is verified. As long as the DLL succeeds in doing that, the punctual code is locked on the incoming signal. Usually, the phase of the code generator is passed through a low-pass filter to generate a code-phase measurement at a user-defined output rate. This filter generally consists of a quadratic fit of the raw code phase over the output measurement interval.

The implementation above gives the correct tracking point in absence of multipath. However, when multipath is present, the correlations represented in Fig. 2 do not apply any more. The I_0, I_{-1} and I_{+1} correlation values are the sum of the correlation of the local code with the incoming LOS signal and of the correlation of the local code with the incoming multipath signal. The result is that the correlation profile is distorted. Fig. 3 illustrates the distortion of the correlation functions in the case of the presence of one multipath signal having a delay of 60 m, a fourth of the power of the direct signal, and being in phase with the direct signal.

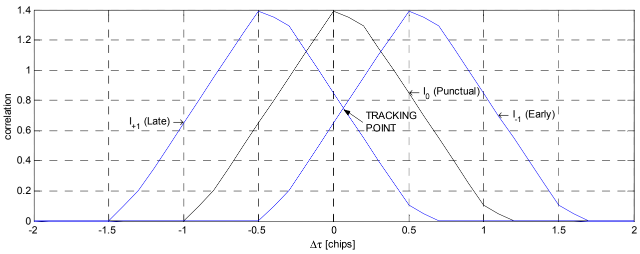

Fig. 3 Distortion of the correlation peak in presence of one multipath signal.

It can be seen that the tracking point, defined by the equality $I_{+1} = I_{-1}$, is no longer positioned at $\Delta\tau = 0$. This means that the DLL is not able to correctly align the punctual code and the incoming code. There is a tracking error, which in Fig. 3 is around 0.1 chips. This tracking error results in an equivalent range error of 29.3 m.

In the general case, the multipath error on the code-phase measurement depends upon the amplitude, delay and phase of the multipath signals. Generally, the maximum error is reached when the multipath signal is in phase with the line-of-sight, and the minimum error when it is in opposition (i.e. 180° out of phase). For all other phase differences, the error would fall in between. To assess the performance of a multipath mitigation method, one often plots the maximum and minimum code error as a function of the multipath delay for a given multipath amplitude.

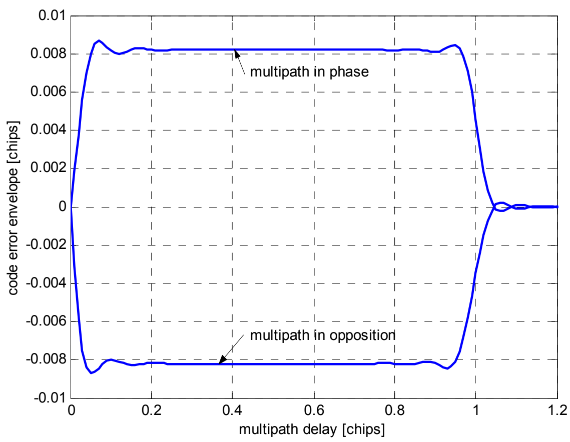

Fig. 4 Multipath error envelope for a narrow correlator DLL (d=1/15, SMR=12dB).

For example, the multipath error envelope as a function of the multipath delay for a narrow correlator DLL is represented in Fig. 4 for a Signal-to-Multipath power Ratio (SMR) of 12dB (amplitude ratio of 4). This plot and all the following plots in this paper have been computed using an Early-Late spacing of d=1/15 chips, and a signal bandwidth of 20 MHz.

GENERAL PRINCIPLE OF THE APME METHOD

Fig. 5 illustrates the modifications to the conventional DLL structure in order to implement the APME technique. The components printed in bold are the additional components with respect to the conventional loop shown in Fig. 1.

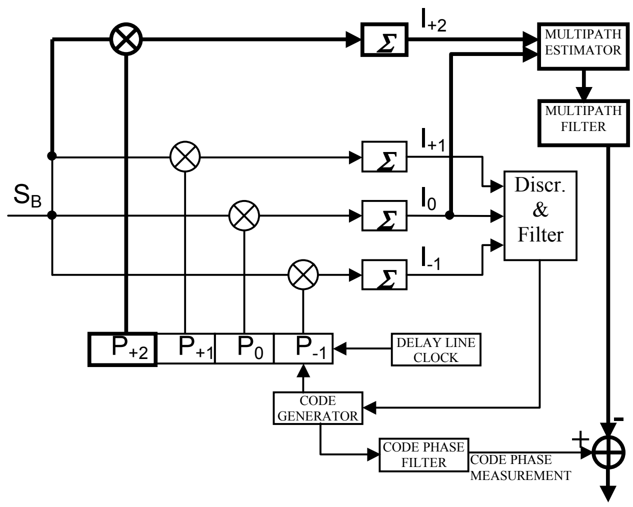

Fig. 5 Implementation of the APME technique.

As mentioned in the introduction, a conventional narrow-spacing DLL is used for the tracking. For the purpose of the multipath mitigation, it will be seen in the next section that an additional correlation value (I_{+2}) is needed. It is generated from a fourth replica of the PRN code, delayed by d with respect to the punctual replica.

The multipath errors are estimated independently from the tracking in a multipath estimator module, on the basis of the I_0 and I_{+2} correlation values. The multipath estimation process has no impact on the tracking process, which keeps the performances of a narrow-correlator loop (low noise). The multipath estimation is performed at every integration step.

The estimation is filtered independently from the range measurement in a low-pass filter. This separate filtering allows to use a low noise equivalent bandwidth B_n, and hence to keep the noise on the estimate low. A typical noise equivalent bandwidth for this filter is 0.1 Hz (single-sided). This small value makes sense because multipath errors typically only contain very low frequency components.

The output of the low-pass filter is an estimation of the multipath error affecting the narrow correlator. It has to be subtracted from the code-phase measurement to yield a corrected measurement where multipath errors are substantially reduced.

The next section provides some insight into the operation principle of the multipath estimator in Fig. 5.

MULTIPATH ESTIMATOR OPERATION

The starting point of the derivation is the well-known property that the signal amplitude measurement delivered by a GPS receiver (or equivalently the C/N0) is highly correlated with multipath errors on the code measurement [5]. This is illustrated by the figure 6, which presents an example of code multipath error and the corresponding signal amplitude. It is clear from Fig. 6 that one could build a good code multipath estimator by properly scaling the signal amplitude, and by removing its mean value. A big advantage of this estimator is that it would operate even for the shortest multipath delays, since it has been demonstrated that the signal amplitude is most sensitive to short multipath delays [5].

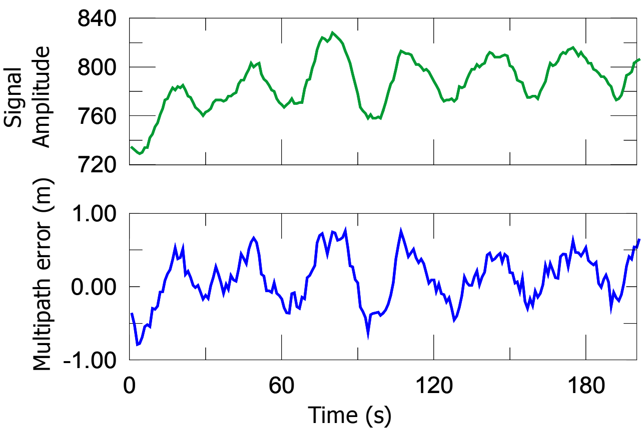

Fig. 6 Code multipath error (lower panel) and corresponding signal amplitude (upper panel).

So far, this property has not been exploited much, mainly because the proportionality factor that links the signal amplitude and the code error is generally unknown: it is a function of the multipath delay and amplitude, which are unknown unless a detailed description of the multipath environment of the antenna is available.

The signal amplitude reported by a GPS receiver is the reading of the punctual correlation value. However, the signal amplitude could equivalently be computed on the basis of any side correlators I_l (l being any integer), at a delay $ld/2$ from the punctual correlation, using the following formula:

$$A_l = \frac{\gamma_l I_l}{1-|l|d/2} \qquad (1)$$

where an appropriate factor has been applied to I_l to compensate for the triangular shape of the correlation peak. For an infinite bandwidth case, this scaling factor would be $1/(1-|l|d/2)$. For a finite bandwidth, it is $\gamma_l/(1-|l|d/2)$ where γ_l is close to 1 and accounts for the rounding of the correlation peak due to the limited signal bandwidth.

All of these estimators of the signal amplitude yield the same result in case no multipath is present, i.e. when the correlation peak is not distorted by any multipath signals.

When multipath enters the receiver, each of the signal amplitude estimates is corrupted by a multipath error, but the error is not the same for all the estimates. It was found that a good estimate of the multipath error of a narrow-correlator tracking loop can be obtained by using an appropriate function of the signal amplitude measured from the punctual correlation (A_0), and from a late correlation at a delay of d with respect to the punctual correlation (A_{+2}). Hence the need for the additional correlator in Fig. 5.

To support the idea, figure 7 shows the normalized signal amplitude computed from I_0 and I_{+2} as a function of the multipath delay, for a SMR of 12dB and for the two cases of a multipath in phase and in opposition. We focus on these two cases because they correspond to the largest multipath error, as was mentioned before. The normalization factor is the signal amplitude of the line-of-sight signal only ($A_{0,LOS}$).

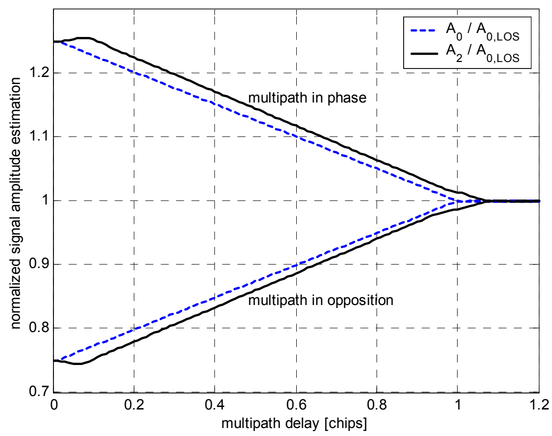

Fig. 7 Normalized signal amplitude computed from I_0 and from I_{+2}.

By looking at Fig. 7, it appears that the difference between $A_{+2}/A_{0,LOS}$ (solid curve) and $A_0/A_{0,LOS}$ (dashed curve) resembles the multipath error in Fig. 4. Namely it is zero for a multipath delay $\delta m=0$ and $\delta m>1.1$ chips, and it does not vary much in the range $0.1<\delta m<1$ chips.

Fig. 8 presents this difference, scaled by 0.42 (see the dashed curve). 0.42 is the scaling factor that yields the best “resemblance” in the least-square sense.

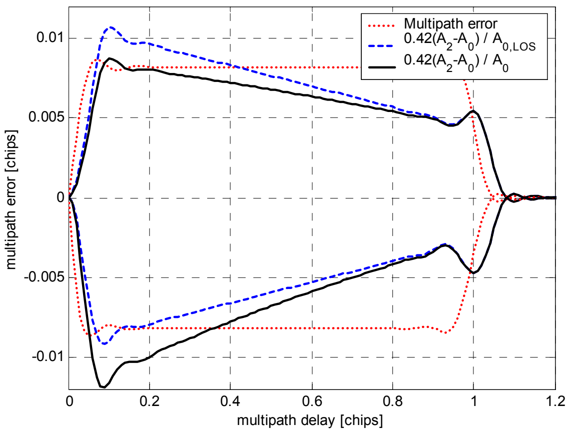

Fig. 8 Multipath error (dotted line), and estimation of it (dashed and solid line).

The quantity $0.42(A_{+2}\text{-}A_0)$ / $A_{0,LOS}$ appears to constitute a good estimate of the actual error (dotted line). The agreement between actual error and its estimate is best for short delays. The applicability of this formula in a real-life situation is limited because $A_{0,LOS}$ is not known by the receiver. However, as shown by the solid line in Fig. 8, using A_0 instead of $A_{0,LOS}$ does not change the conclusion.

The previous example indicates that the following quantity accounts for an important part of the multipath error affecting a narrow correlator DLL:

$$MP = 0.42\frac{A_{+2} - A_0}{A_0} \tag{2}$$

where MP is in units of chips. Using (1), (2) can be rewritten as:

$$MP = -0.42 \cdot \left(1 - \frac{\gamma_{+2} I_{+2}}{\gamma_0 I_0}\frac{1}{1-d}\right) \tag{3}$$

To convert MP in units of length, it has to be multiplied by the code wavelength λ_c, which is 293 m for the GPS CA-code.

Equation (3) was deduced from the particular case of a SMR of 12 dB, and a multipath phase of 0 and 180°. The figures 9 and 10 demonstrate that Equation (3) is equally able to estimate an important amount of the multipath error for other values of the SMR and of the multipath phase respectively, apart from a large negative overshoot for a SMR of 6 dB and a delay around 0.1 chips.

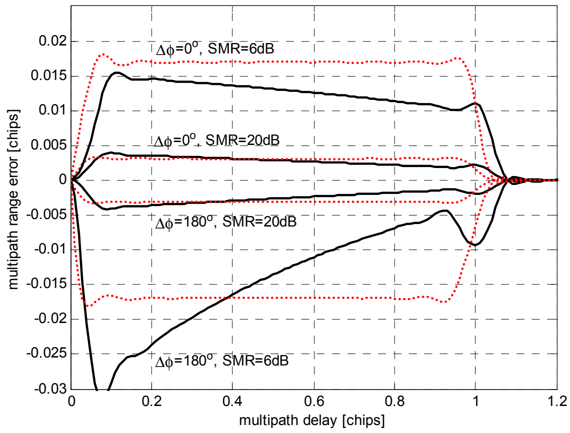

Fig. 9 Actual error (dotted line) and estimation thereof from Equation (3) for an SMR of 6dB and 20dB.

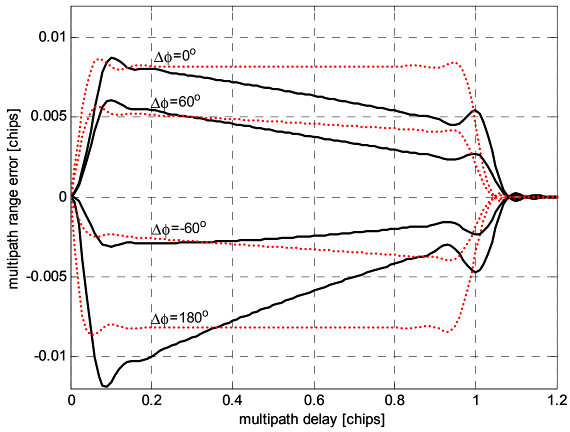

Fig. 10 Actual error (dotted line) and estimation thereof from Equation (3) for different value of the multipath phase, and SMR=12dB.

To refine the agreement between the multipath estimation and the actual error, it could be tempting to use more signal amplitudes than I_0 and I_{+2}. One could for example imagine that the multipath error is estimated by M early and N late correlators, and generalize Equation (3) to:

$$MP = \sum_{l=-M..N} \alpha_l \frac{1}{\gamma_0 I_0} \frac{\gamma_l I_l}{1-|l|\frac{d}{2}} \tag{4}$$

The α_l coefficients are chosen such to make the estimate, MP, best fit the actual error. Tests and simulations have shown that increasing the number of correlations participating to the estimation results in being able to better fit the multipath error, at the expense of receiver complexity and noise on the multipath estimate. Equation (3) represents the best compromise between accuracy, receiver complexity and noise on the estimation (see Appendix for a derivation of the noise characteristics of

MP). Therefore, the multipath estimator module in Fig. 5 computes its output according to Equation (3).

THEORETICAL PERFORMANCES

The performance of the APME technique can be evaluated by plotting the resulting code error envelope, when subtracting the estimated error from the code-phase measurement. Figures 11 to 13 represent multipath error envelopes for three different signals to multipath power ratios (20dB, 12dB and 6dB). In each plot, three multipath mitigation techniques are compared: the narrow correlator technique, the strobe correlator technique (using a bipolar symmetrical strobe [9]), and the APME technique. Other state-of-the-art techniques offer similar or worse performances as the strobe correlator. All the techniques were simulated on MATLAB using an early-late spacing of *d*=1/15 chips and a signal bandwidth of 20 MHz.

It is clear from all the plots that the APME technique provides the best performances for short to very short multipath delays (delays lower than 20 m or 0.07 chips), with up to 50% improvement in the high SMR case. For larger delays, the APME method provides a substantial improvement with respect to the narrow correlator technique, but is less efficient than the strobe correlator. The APME technique also exhibits better performances for SMR higher than 12dB (corresponding to an amplitude ratio of 4). It is expected that most of the real-life multipath will fall in this range, especially if a choke ring antenna, or equivalent, is used.

It is a common misunderstanding that the total area inside the multipath error envelope is an indication of the quality of the multipath mitigation. However, as will be shown in the following, the APME technique performs better than the strobe correlator, although the total error area of the latter is much smaller. The reason is that real-life GPS signals tend to be dominated by short to very short delay multipath. This means that the key parameter to assess the effectiveness of a multipath mitigation scheme is its area at short delays, and **not** its total area.

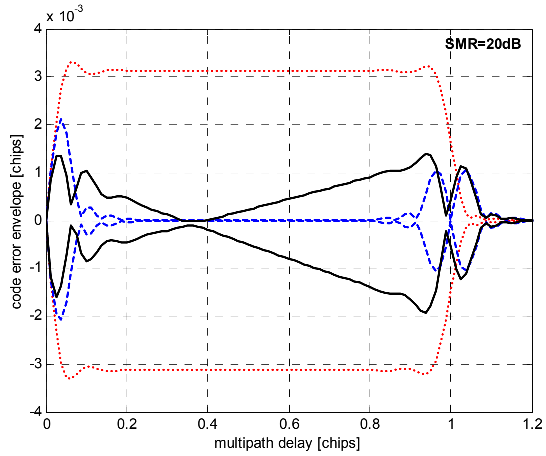

Fig. 11 Code error envelope for the narrow correlator (dotted red), the strobe correlator (dashed blue) and the APME (solid black) for an SMR of 20dB.

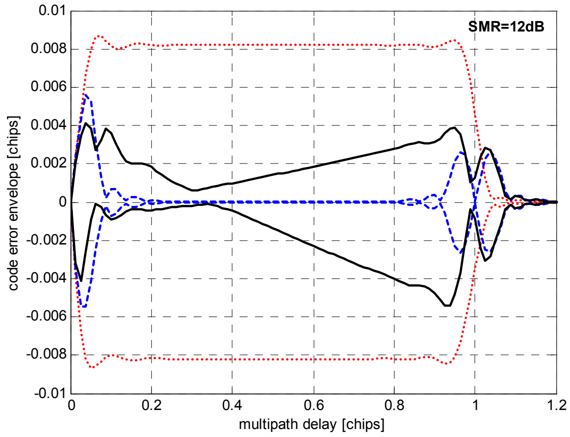

Fig. 12 Code error envelope for the narrow correlator (dotted red), the strobe correlator (dashed blue) and the APME (solid black) for an SMR of 12dB.

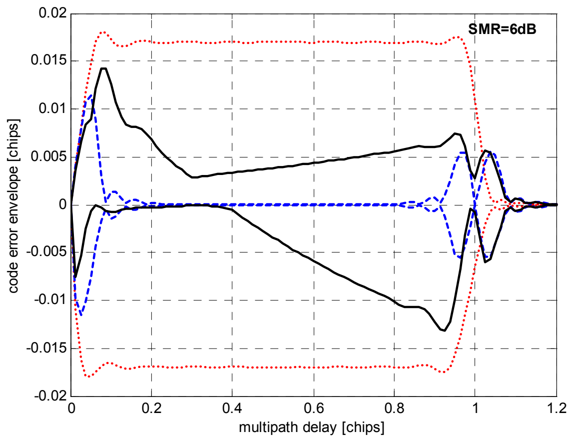

Fig. 13 Code error envelope for the narrow correlator (dotted red), the strobe correlator (dashed blue) and the APME (solid black) for an SMR of 6dB.

NOISE COMPUTATION

So far, we have focused on the quality of the multipath error estimation from an accuracy point of view. However, it is preferred that the noise on the multipath estimation is small, because this noise is added to the code-phase measurement noise when subtracting the multipath estimation from the code-phase measurement.

It is shown in the Appendix that the standard deviation of the noise on the multipath estimation is given by (in units of meters):

$$\sigma_{MP} = 0.42\lambda_c \sqrt{B_n \frac{N_0}{S}} \sqrt{\frac{d(2-d)}{(1-d)^2}} \qquad (5)$$

This amounts to 0.085 m for λ_c=293m, B_n=0.1Hz, d=1/15 and a nominal C/N0 of 45dB-Hz.

Typically, the noise standard deviation of the code-phase measurement is in the order of 0.2 m. Applying the APME method increases this value to $\sqrt{0.2^2 + 0.085^2}$ =0.217 m, or by a factor of only 10%. This small penalty is negligible in comparison with the large reduction of the multipath error. As a comparison, the Strobe or the HRC techniques result in an increase of the noise by 40% [3].

FIELD RESULT

To evaluate the effectiveness of the APME technique in a real-life situation, we analysed 24 hours of data at a fixed location on a roof in the center of Leuven (Belgium). This location is prone to multipath signals originating from other buildings at distances ranging from 20 to 100m. In order to compare the results with other state-of-the-art techniques, we built a zero baseline with a reference receiver implementing the strobe correlation technique. The two receivers were connected to the same antenna through a power splitter and hence experienced the same multipath signals. The antenna was a Sensor Systems S67-1575-96, not specially designed for multipath mitigation (no choke ring or equivalent).

The first receiver delivered measurements from the narrow-correlator tracking (d=1/15), and corrected measurements after applying the APME scheme. The reference receiver delivered measurements from the strobe correlator tracking. Both receivers output their measurement at a 1-Hz rate and used a code-smoothing interval of 10 s. The main effect of such a short-interval smoothing is to reduce the measurement noise while leaving most of the multipath error unchanged. The elevation mask was set to 0°.

For all the satellite passes during the 24-hour period, C/A-code multipath and noise was extracted from the data using the well-known multipath combination of code and phase measurements [5]. In total more than 180 hours of common data were analysed, ensuring that many different kinds of multipath signals were included in the data. Actually taking more than 1 day of data would not be useful due to the day-to-day repeatability of the multipath errors.

Fig. 14 illustrates the standard deviation of the multipath and noise for the three mitigation techniques (narrow and strobe correlators, and APME), for each of the satellites available that day. Fig. 15 presents the same result but the values have been normalized by the standard deviation of the error affecting the narrow correlator.

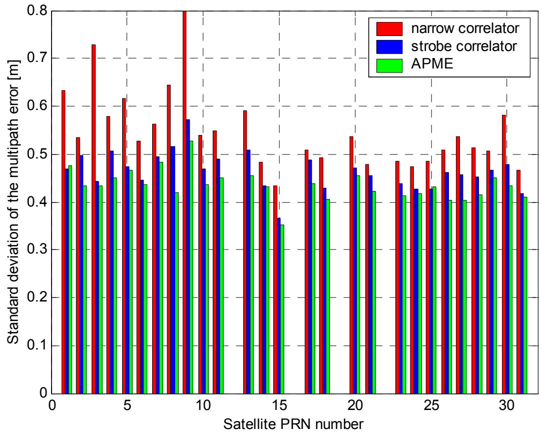

Fig. 14 Comparison of multipath error standard deviation over a 24-hour period.

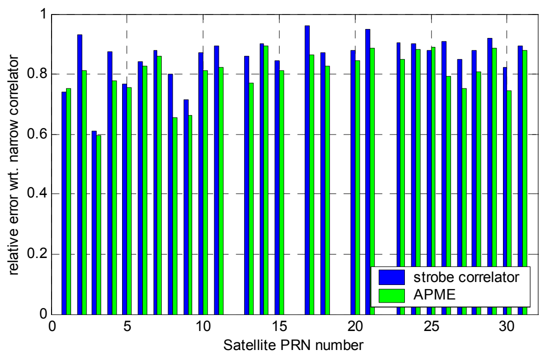

Fig. 15 Relative multipath error with respect to Narrow Correlator over a 24-hour period.

It can be seen that the strobe correlator yields an average multipath-noise reduction by 15% with respect to the narrow correlator, and the APME offers 5% additional improvement on average. In our data set, the APME performed better than the strobe correlator for 25 out of 27 satellites.

Fig. 16 to 18 present real data taken during this experiment. The data were chosen to illustrate a case where the APME technique performed worse than the strobe correlator (Fig. 16), equivalently (Fig. 17) and better (Fig. 18). In all the plots, the error affecting the narrow correlator tracking is plotted in red, the estimation thereof using Equation (3) in black and the corrected code-phase measurement error in green. The code-phase error from the reference receiver using the strobe correlator technique is in blue. For clarity, the curves have been vertically shifted apart.

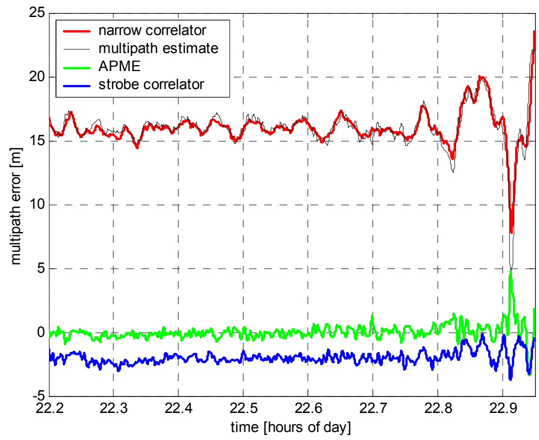

Fig. 16 Comparison of multipath errors on the narrow correlator, the APME and the strobe correlator.

In Fig. 16, a strong multipath signal affects the tracking at time 22.9 hours. As was predicted by the theory, the multipath estimation overestimates the negative part of the multipath error (see Fig. 9). As a result, the corrected range exhibits a positive error. However, this error is much smaller than the original narrow correlator error.

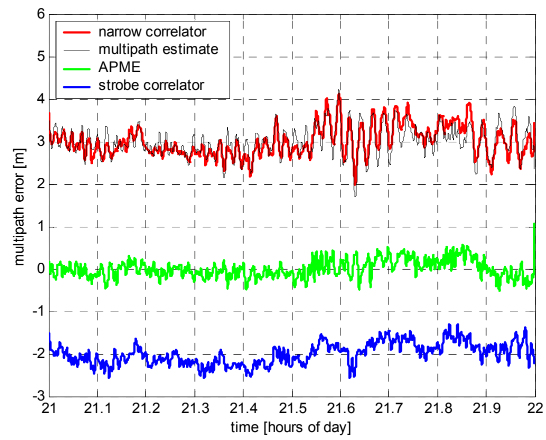

Fig. 17 Comparison of multipath errors on the narrow correlator, the APME and the strobe correlator.

Fig. 17 illustrates a typical mild multipath perturbation. The narrow correlator error exhibits the characteristic oscillating behavior, with peak-to-peak amplitude of around 2 m. In this case both the APME and the strobe correlator performed equivalently good, and suppressed a large part of the multipath perturbation.

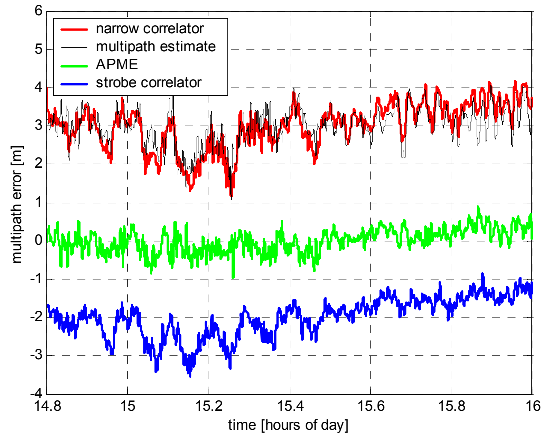

Fig. 18 Comparison of multipath errors on the narrow correlator, the APME and the strobe correlator.

Finally, Fig. 18 illustrates a typical case where long-period multipath corrupts both the narrow and the strobe correlator tracking. In this case, the APME method performs much better than the other ones. This is in accordance with the rule of thumb that long-period multipaths are often associated with short delays, where the APME approach is the most powerful.

A more complete analysis of the 24-hour data files showed that most of the real-life multipath was of the kind illustrated by either Fig. 17 or 18.

FURTHER PROSPECTS

The last section demonstrated the high potential of the APME technique. Further work will be focused on extending its capabilities and performances. In particular, the following points will be tackled:

- Preliminary tests have shown that the estimation overshoot in case of low SMR could be limited by using an adaptive version of the APME (AAPME). The basic idea would be to continuously adapt the 0.42 coefficient such that the error estimate best fits the actual multipath error as computed form the code-phase combination.
- The APME has been demonstrated for a wide-band signal like the 20 MHz-wide GPS signal. It is expected that a similar technique can be tuned for the narrow band case, like WAAS (2.2 MHz-wide).
- The APME has proved very powerful for code multipath mitigation. Further work is needed to assess the potential of a similar approach for carrier phase multipath.

CONCLUSION

State-of-the-art technologies for mitigating multipath achieve a dramatic reduction of the total area of the multipath error envelope by annihilating all multipath components of medium to large delay. However, they are inefficient against close-in, short-delay multipath. This is a serious limitation since real GPS data tend to be dominated by the short-delay case.

This paper presented a new way of performing multipath mitigation, which achieves superior performances against short-delay multipath. The technique relies on an a-posteriori estimation of the error affecting the code tracking loop. The originality of the method lies in its use of the signal amplitude measurements, which are known to be correlated with the code error, especially for short-delay multipath.

Compared to previous techniques, this method achieves better performances against the prevailing short-delay multipath (<20m delay) while relaxing the performances for the rare medium and large delays.

Although the total area of the resulting error envelope is larger than what is achieved with other techniques, a consistent improvement with respect to state-of-the-art technologies has been demonstrated from real GPS data, confirming that the dominant multipath are indeed of short-delay type. This demonstrates that the performance of a multipath mitigation technology should be evaluated from its short-delay behavior, and not from its error envelope area.

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APPENDIX

This Appendix provides a derivation of the noise standard deviation of the multipath estimate MP given by Equation (3). For simplicity, we make the approximation that the signal is of infinite bandwidth (γ_l=1) and that no multipath is present.

The variance of the noise of MP computed from (3) is given by (expressed in chips^2):

$$\sigma_{MP}^2 = 0.42^2 \frac{1}{(1-d)^2} \sigma^2 (\frac{I_{+2}}{I_0}) \qquad \text{(A.1)}$$

where I_{+2} and I_0 are two gaussian random variable with the following mean, variance and covariance properties [7]:

$$\begin{aligned} \mu_{I_{+2}} &= \sqrt{2\frac{S}{N_0}T}(1-d) \\ \mu_{I_0} &= \sqrt{2\frac{S}{N_0}T} \\ \sigma_{I_{+2}}^2 &= 1 = \sigma_{I_0}^2 \\ COV_{I_{+2},I_0} &= 1-d \end{aligned} \qquad \text{(A.2)}$$

where T is the accumulation duration (typically 10 ms), S is the received signal power (typ. -130dBm), and N_0 is the power spectral density of the noise (typ. -204dBW/Hz).

I_{+2} and I_0 can be rewritten as μ_{+2} + n_{+2} and μ_0 + n_0 respectively, with n_{+2} and n_0 accounting for the noise on the correlation values. The variance of n_{+2} and n_0 is $E[n_{+2}^2] = E[n_0^2] = 1$, and their covariance is $E[n_{+2}n_0] = 1-d$.

By noting that n_0 is much smaller than μ_0 in normal situations, I_{+2}/I_0 can be approximated to the second order as:

$$\begin{aligned} \frac{I_{+2}}{I_0} &= \frac{\mu_{+2}+n_{+2}}{\mu_0+n_0} \\ &\approx \frac{\mu_{+2}+n_{+2}}{\mu_0}(1-\frac{n_0}{\mu_0}+\frac{n_0^2}{\mu_0^2}) \end{aligned} \qquad \text{(A.3)}$$

After some algebraic manipulation, it can be derived that the variance of I_{+2}/I_0 can be approximated to the second order as:

$$\sigma^2(\frac{I_{+2}}{I_0}) = (\frac{\mu_{I_{+2}}}{\mu_{I_0}})^2 \left(\frac{\sigma_{I_{+2}}^2}{\mu_{I_{+2}}^2} + \frac{\sigma_{I_0}^2}{\mu_{I_0}^2} - 2\frac{COV_{I_{+2},I_0}}{\mu_{I_{+2}}\mu_{I_0}} \right) \qquad \text{(A.4)}$$

which reduces to the following when the definitions (A.2) are used.

$$\sigma^2(\frac{I_{+2}}{I_0}) = \frac{N_0}{2ST} d(2-d) \qquad \text{(A.5)}$$

By inserting (A.5) into (A.1), the standard deviation of the multipath estimate before the low-pass filter can be computed:

$$\sigma_{MP} = 0.42\sqrt{\frac{1}{2T}\frac{N_0}{S}}\sqrt{\frac{d(2-d)}{(1-d)^2}} \qquad \text{(A.6)}$$

where σ_{MP} is expressed in chips. After the low-pass filter having a single-sided noise equivalent bandwidth of B_n, σ_{MP} becomes:

$$\sigma_{MP} = 0.42\sqrt{B_n\frac{N_0}{S}}\sqrt{\frac{d(2-d)}{(1-d)^2}} \qquad \text{(A.7)}$$